Improving Offensive Performance through Opponent Modeling

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Abstract

Although in theory opponent modeling can be useful in any adversarial domain, in practice it is both difficult to do accurately and to use effectively to improve game play. In this paper, we present an approach for online opponent modeling and illustrate how it can be used to improve offensive performance in the Rush 2008 football game. In football, team behaviors have an observable spatio-temporal structure, defined by the relative physical positions of team members over time; we demonstrate that this structure can be exploited to recognize football plays at a very early stage of the play using a supervised learning method. Based on the teams' play history, our system evaluates the competitive advantage of executing a *play switch* based on the potential of other plays to increase the yardage gained and the similarity of the candidate plays to the current play. In this paper, we investigate two types of play switches: 1) whole team and 2) subgroup switching. Both types of play switches improve offensive performance, but by only modifying the behavior of a key subgroup of offensive players, we improve on the yardage gained.

Introduction

By accessing the play history of your opponent, it is possible to glean critical insights about future plays. This was recently demonstrated at a soccer match by an innovative, well-prepared goalkeeper who used his iPod to review a video play history of the player taking a penalty kick; identifying the player's tendency to kick to the left allowed the goalkeeper to successfully block the shot (Bennett 2009). Although play history can be a useful source of information, it is difficult to utilize effectively in a situation with a large number of multi-agent interactions. Opponent modeling can be divided into three categories: 1) online tracking 2) online strategy recognition and 3) off-line review. In online tracking, immediate future actions of individual players (passes, feints) are predicted, whereas in online strategy recognition, the observer attempts to recognize the high-level strategy used by the entire team. In offline review, general strengths, weaknesses, and tendencies are identified in an offline setting and used as part of the training/learning regimen.

This paper addresses the problem of online strategy recognition in adversarial team games. In physical domains

Figure 1: Screenshot of the Rush 2008 football simulator. The offense team (shown in red) is using the play split 8 and being countered by the defense (shown in blue) using a 31 formation (variant 1).

(military or athletic), team behaviors often have an observable spatio-temporal structure, defined by the relative physical positions of team members. This structure can be exploited to perform behavior recognition on traces of agent activity over time. This paper describes a method for recognizing defensive plays from spatio-temporal traces of player movement in the Rush 2008 football game (see Figure 1) and using this information to improve offensive play.

To succeed at American football, a team must be able to successfully execute closely-coordinated physical behavior. To achieve this tight physical coordination, teams rely upon a pre-existing playbook of offensive maneuvers to move the ball down the field and defensive strategies to counter the opposing team's attempts to make yardage gains. Rush 2008 simulates a modified version of American football; plays in Rush are composed of a starting formation and instructions for each player in the formation. These instructions are similar to a conditional plan and include choice points where the players can make individual decisions as well as pre-defined behaviors that the player executes to the best of its physical capability. Rush 2008 was developed from the open source

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14. ABSTRACT

Although in theory opponent modeling can be useful in any adversarial domain, in practice it is both difficult to do accurately and to use effectively to improve game play. In this paper, we present an approach for online opponent modeling and illustrate how it can be used to improve offensive performance in the Rush 2008 football game. In football, team behaviors have an observable spatio-temporal structure, defined by the relative physical positions of team members over time we demonstrate that this structure can be exploited to recognize football plays at a very early stage of the play using a supervised learning method. Based on the teams? play history our system evaluates the competitive advantage of executing a play switch based on the potential of other plays to increase the yardage gained and the similarity of the candidate plays to the current play. In this paper, we investigate two types of play switches: 1) whole team and 2) subgroup switching. Both types of play switches improve offensive performance but by only modifying the behavior of a key subgroup of offensive players, we improve on the yardage gained.

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Rush 2005 game, which is similar in spirit to Tecmo Bowl and NFL Blitz (Rush2005 2005).

Although there have been other studies examining the problem of recognizing completed football plays, we present results on recognizing football plays online at an early stage of play, and demonstrate a mechanism for exploiting this knowledge to improve a team's offense. Our system evaluates the competitive advantage of executing a *play switch* based on the potential of other plays to improve the yardage gained and the similarity of the candidate plays to the current play. Our play switch selection mechanism outperforms both the built-in offense and a greedy yardage-based switching strategy. Calculating the relative similarity of the current play compared to the proposed play is shown to be a necessary step to reduce confusion on the field and effectively boost performance. Additionally we investigated the utility of limiting the play switch to a small subgroup of players; by only modifying the actions of small subgroup of key players, we can improve on the total team switch.

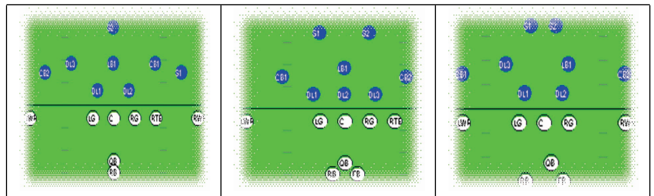
Related Work

Previous work on team behavior recognition has been primarily evaluated within athletic domains, including American football (Intille and Bobick 1999), basketball (Bhandari *et al.* 1997; Jug *et al.* 2003), and Robocup soccer simulations (Riley and Veloso 2000; 2002; Kuhlmann *et al.* 2006). To recognize athletic behaviors, researchers have exploited simple region-based (Intille and Bobick 1999) or distance-based (Riley and Veloso 2002) heuristics to build accurate, but domain-specific classifiers. For instance, based on the premise that all behaviors always occur on the same playing field with a known number of entities, it is often possible to divide the playing field into grids or typed regions (e.g., goal, scrimmage line) that can be used to classify player actions. In contrast, we train our classifiers on raw observation traces and do not rely on a field-based marker system.

In Robocup, there has been some research on team intent recognition geared towards the Robocup coach competition. Techniques have been developed to extract specific information, such as home areas (Riley *et al.* 2002), opponent positions during set-plays (Riley and Veloso 2002), and adversarial models (Riley and Veloso 2000), from logs of Robocup simulation league games. This information can be utilized by the coach agent to improve the team's scoring performance. For instance, information about opponent agent home areas can be used triggers for coaching advice and for doing "formation-based marking", in which different team members are assigned to track members of the opposing team. However, the focus of the coaching agents is to improve performance of teams in future games; our system immediately takes action on the recognized play to evaluate possible play switches.

Rush Football

Football is a contest of two teams played on a rectangular field that is bordered on lengthwise sides by an end zone. Unlike American football, Rush teams only have 8 players on the field at a time out of a roster of 18 players. and the field is 100 yards by 63 yards. The game's objective is to out-score the opponent, where the offense (i.e., the team with possession of the ball), attempts to advance the ball from the line of scrimmage into their opponent's end zone. In a full game, the offensive team has four attempts to get a *first down* by moving the ball 10 yards down the field. If the ball is intercepted or fumbled, ball possession transfers to the defensive team.



Pro vs 23	Power vs 31	Split vs 2222

Figure 2: Three offensive and defensive configurations. Offensive players are shown in white and the defense in blue.

A Rush play is composed of (1) a starting formation and (2) instructions for each player in that formation. A formation is a set of (x,y) offsets from the center of the line of scrimmage. By default, directions for each player consist of (a) an offset/destination point on the field to run to, and (b) a behavior to execute when they get there. Play instructions are similar to a conditional plan and include choice points where the players can make individual decisions as well as pre-defined behaviors that the player executes to the best of their physical capability. Rush includes three offensive formations (**power**, **pro**, and **split**) and four defensive ones (**23**, **31**, **2222**, **2231**) 2. Each formation has eight different plays (numbered 1-8) that can be executed from that formation. Offensive plays typically include a handoff to the running back/fullback or a pass executed by the quarterback to one of the receivers, along with instructions for a running pattern to be followed by all the receivers.

Play Recognition using SVM

In this paper we focus on intent recognition from the viewpoint of the offense: given a series of observations, our goal is to recognize the defensive play as quickly as possible in order to maximize our team's ability to intelligently respond with the best offense. Thus, the observation sequence grows with time unlike in standard offline activity recognition where the entire set of observations is available. We approach the problem by training a series of multi-class discriminative classifiers, each of which is designed to handle observation sequences of a particular length. In general, we expect that the early classifiers should be less accurate since they are operating with a shorter observation vector and because the positions of the players have deviated little from the initial formation.

We perform this classification using support vector machines (Vapnik 1998). Support vector machines (SVM) are a supervised binary classification algorithm that have been demonstrated to perform well on a variety of pattern classification tasks, particularly when the dimensionality of

the data is high (as in our case). Intuitively the support vector machine projects data points into a higher dimensional space, specified by a kernel function, and computes a maximum-margin hyperplane decision surface that separates the two classes. Support vectors are those data points that lie closest to this decision surface; if these data points were removed from the training data, the decision surface would change. More formally, given a labeled training set $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_l, y_l)\}$, where $\mathbf{x}_i \in \Re^N$ is a feature vector and $y_i \in \{-1, +1\}$ is its binary class label, an SVM requires solving the following optimization problem:

$$\min_{\mathbf{w},b,\xi} \frac{1}{2}\mathbf{w}^T\mathbf{w} + C\sum_{i=1}^{l}\xi_i$$

constrained by:

$$\begin{aligned} y_i(\mathbf{w}^T\phi(\mathbf{x}_i) + \mathbf{b}) &\geq 1 - \xi_i, \\ \xi_i &\geq 0. \end{aligned}$$

The function $\phi(.)$ that maps data points into the higher dimensional space is not explicitly represented; rather, a *kernel* function, $K(\mathbf{x}_i, \mathbf{x}_j) \equiv \phi(\mathbf{x}_i)\phi(\mathbf{x}_j)$, is used to implicitly specify this mapping. In our application, we use the popular radial basis function (RBF) kernel:

$$K(x_i, x_j) = \exp(-\gamma||x_i - x_j||^2), \gamma > 0.$$

Several extensions have been proposed to enable SVMs to operate on multi-class problems (with k rather than 2 classes), such as one-vs-all, one-vs-one, and error-correcting output codes. We employ a standard one-vs-one voting scheme where all pairwise binary classifiers, $k(k-1)/2 = 28$ for every multi-class problem in our case, are trained and the most popular class is selected. When multiple classes receive the highest vote, we select the winning one with the lowest index; the benefit of this approach is that classification is deterministic but it can bias our classification in favor of lower-numbered plays. For a real game system, we would employ a randomized tie-breaking strategy. Many efficient implementations of SVMs are publicly available; we use LIBSVM (Chang and Lin 2001).

We train our classifiers using a collection of simulated games in Rush collected under controlled conditions: 40 instances of every possible combination of offense (8) and defense plays (8), from each of the 12 starting formation configurations. Since the starting configuration is known, each series of SVMs is only trained with data that could be observed starting from its given configuration. For each configuration, we create a series of training sequences that accumulates spatio-temporal traces from $t = 0$ up to $t \in \{2, \ldots, 10\}$ time steps. A multiclass SVM (i.e., a collection of 28 binary SVMs) is trained for each of these cases. Although the aggregate number of binary classifiers is large, each classifier only employs a small fraction of the dataset and is therefore efficient (and highly paralellizable). Cross-validation was used to tune the SVM parameters (C and σ) for all of the SVMs.

Classification at testing time is very fast and proceeds as follows. We select the multiclass SVM that is relevant to the current starting configuration and time step. An observation vector of the correct length is generated (this can be done incrementally during game play) and fed to the multi-class SVM. The output of the intent recognizer is the system's best guess (at the current time step) about the opponent's choice of defensive play and can help us to select the most appropriate offense, as discussed below.

Table 1 summarizes the experimental results for different lengths of the observation vector (time from start of play), averaging classification accuracy across all starting formation choices and defense choices. We see that at the earliest timestep, our classification accuracy is at the baseline but jumps sharply near perfect levels at $t = 3$. This strongly confirms the feasibility of accurate intent recognition in Rush, even during very early stages of a play. At $t = 2$, there is insufficient information to discriminate between offense plays (perceptual aliasing), however by $t = 3$, the positions of the offensive team are distinctive enough to be reliably recognized.

Offensive Play Switches

To improve offensive performance, our system evaluates the competitive advantage of executing a *play switch* based on 1) the potential of other plays to improve the yardage gained and 2) the similarity of the candidate plays to the current play. First, we train a set of SVM models to recognize defensive plays at a particular time horizon as described in the previous section; this training data is then used to identity promising play switches. A play switch is executed:

1. after the defensive play has been identified by the SVM classifier;
2. if there is a stronger alternate play based on the yardage history of that play vs. the defense;
3. if the candidate play is sufficiently similar to the current play to be feasible for immediate execution.

To determine whether to execute the play switch for a particular combination of plays, the agent considers N, the set of all offensive plays shown to gain more than a threshold ϵ value. The agent then selects $Min(n \in N)$, the play in the list most like the current play for each play configuration and caches the preferred play in a lookup table.

When a play is executed, the agent will use all observations up to and including observation 3 to determine what play the defense is executing before performing a lookup to determine the play switch to make. The process is ended with execution of a change order to all members of the offensive team. Calculating the feasibility of the play switch based on play similarity is a crucial part of improving the team's performance; in the results section, we evaluate our similarity-based play switch mechanism vs. a greedy play switching algorithm that focuses solely on the potential for yardage gained.

Play Similarity Metric

To calculate play similarities, we create a feature matrix for all offensive formation/play combinations based on the training data.

Table 1: Play recognition results (accuracy over all play combinations)

$t = 2$	3	4	5	6	7	8	9	10
12.50	96.88	96.87	96.85	96.84	96.87	96.89	96.83	96.81

The features collected for each athlete A are

Max(X): The rightmost position traveled to by A

Max(Y): The highest position traveled to by A

Min(X): The leftmost position traveled to by A

Min(Y): The lowest position traveled to by A

Mean(X): $= \frac{\sum_{i=0}^{N-1} X_i}{N}$

Mean(Y): $= \frac{\sum_{i=0}^{N-1} Y_i}{N}$

Median(X): $= Sort(X)_{i/2}$

Median(Y): $= Sort(Y)_{i/2}$

FirstToLastAngle: Angle from starting point $(x1, y1)$, to ending point $(x2, y2)$, is defined as $atan\left(\frac{\triangle y}{\triangle x}\right)$

Start_Angle: Angle from the starting point (x_0, y_0) to (x_1, y_1), defined as $atan\left(\frac{y}{x}\right)$

End_Angle: Angle from the starting point (x_{n-1}, y_{n-1}) to (x_n, y_n), defined as $atan\left(\frac{\triangle y}{\triangle x}\right)$

Total_Angle: $= \sum_{i=0}^{N-1} atan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)$

Total_Path_Distance: $= \sum_{i=0}^{N-1}\left(\sqrt[2]{x_i{}^2 + y_i{}^2}\right)$

Feature set F for a given play c contains all the features for each offensive player in the play and is described as

$$\overrightarrow{F_c} = \{A_{c1} \cup A_{c1} \cup A_{c2} \cup \cdots \cup A_{c8}\}$$

These features are similar to the ones used in (Rubine 1991) and more recently, by (Wobbrock *et al.* 2007) to match pen trajectories in sketch-based recognition tasks, but generalized to handle multi-player trajectories. To compare plays we use the sum of the absolute value of the differences (L_1 norm) between each feature F_{ci}. This information is used to build a similarity matrix M_{ij} for each possible offensive play combination as defined below.

$$M_{ij} = \sum_{c=0}^{\left\|\overrightarrow{F_c}\right\|-1} \Delta\overrightarrow{F_c}$$

$$i, j = 1 \ldots 8$$

There is one matrix M for each offensive formation O_β, where $\beta = \{$pro, power, split$\}$ are the offensive formations. Defensive formation/play combinations are indicated by $D_{\alpha p}$, where $\alpha = \{23,\ 31,\ 2222,\ 2231\}$ and p represents plays 1..8. *M* for a specific play configuration is expressed as $O_\beta D_{\alpha p} M_i$, given *i (1...8)* is our current offensive play. The purpose of this algorithm is to find a value *j* (play) most similar to *i* (our current play), with a history (based on earlier observation) of scoring the most yardage. This process is accomplished for every offensive play formation against every defensive play formation and play combination. When the agent is constructing the lookup table and needs to determine the most similar play from a list, given current play i, it calls the method, $min(O_\beta D_{\alpha p} M_i)$ which returns the most similar play.

Improving the Offense

Our algorithm for improving Rush offensive play has two main phases, a preprocess stage which yields a play switch lookup table and an execution stage where the defensive play is recognized and the offense responds with an appropriate play switch for that defensive play. As described in Section we train a set of SVM classifiers using 40 instances of every possible combination of offense (8) and defense plays (8), from each of the 12 starting formation configurations. This stage yields a set of models used for play recognition during the game. Next, we calculate and cache play switches using the following procedure:

Step 1: Collect data by running the RUSH 2008 football simulator 50 times for every play combination.

Step 2: Create yardage lookup tables for each play combination. This information alone is insufficient to determine how good a potential play is to perform the play switch action on. The transition play must resemble our current offensive play or the offensive team will spend too much time retracing steps and perform very poorly.

Step 3: Create feature matrix for all offensive formation/play combinations using the probabilistic trace representation.

Step 4: Create the final play switch lookup table based on both the yardage information and the play similarity.

To create the play switch lookup table, the agent first extracts a list of offensive plays L given the requirement $yards\,(L_i) > \epsilon$ where ϵ is the smallest yardage gained in which the agent does not consider changing the current offensive play to another. We used $\epsilon = 1.95$ based on a quadratic polynomial fit of total yardage gained in 6 tests with $\epsilon = \{MIN, 1.1, 1.6, 2.1, 2.6, MAX\}$ where MIN is small enough no plays are selected to change and MAX where all plays are selected for change to the highest yardage play with no similarity comparison. Second, from the list L find the play most similar (smallest value in the matrix) to our current play *i* using $Min(O_\beta D_{\alpha p} M_i)$ and add it to the lookup file.

During execution, the offense uses the following procedure:

1. At each observation less than 4, collect movement traces for each play.

2. At observation 3, use LIBSVM with the collected movement traces and previously trained SVM models to identify the defensive play.
3. Access the lookup file to find $best(i)$ for our current play i.
4. Send a change order command to the offensive team to change to play $best(i)$.

However it is not necessary (or always desirable) to change all of the players to the new play. We also investigated the performance of subgroup switching, modifying the play of small group of key players while leaving the remaining players alone. By segmenting the team in this fashion we are able to in essence combine two plays which had previously been identified as alike to each other with regard to spatio-temporal data, but different in regards to yards gained. The football offensive team lends itself to three main groups based on domain knowledge of football. Group 1 contains the QB, RB, and FB; group 2 has LG, C and RG; and group 3 consists of the remaining players LWR, RWR, RTE, and LTE.

Figure 3 is a good example of a successful merging of two plays to produce a superior play given this defense. The green line represents the average yardage gained. The left image is the most likely path of the baseline case (a running play which yields little yardage on average). The middle image is the most likely execution trace produced by the total play switch method. The play produced by the total play switch was not much more successful than the baseline case; however when only Group 1 (QB, RB, FB) is modified the success of the play increases greatly and the new play is shown to be very coordinated and effective.

Empirical Evaluation

The algorithm was tested using the RUSH 2008 simulator for ten plays on each possible play configuration in three separate trials. We compared our play switch model (using the yardage threshold $\epsilon = 1.95$ as determined by the quadratic fit) to the baseline Rush offense and to a greedy play switch strategy ($\epsilon = MAX$) based solely on the yardage.

Overall, the average performance of the offense went from 2.82 yards per play to 3.65 yards per play ($\epsilon = 1.95$) with an overall increase of 29%, $\pm 1.5\%$ based on sampling of three sets of ten trials. An analysis of each of the formation combinations (Figure 6) shows the yardage gain varies from as much as 100% to as little as 0.1%. Overall, performance is consistently better for every configuration tested. In all cases, the new average yardage is over 2.3 yards per play with no weak plays as seen in the baseline. For example, Power vs. 23 (1.4 average yards per play) and Power vs. 2222 (1.3 average yards per play). Results with $\epsilon = MAX$ clearly shows simply changing to the greatest yardage generally results in poor performance from the offense.

Power vs. 23 is dramatically boosted from about 1.5 yards to about 3 yards per play, doubling yards gained. Other combinations, such as Split vs. 23 and Pro vs. 32 already scored good yardage and improved less dramatically at

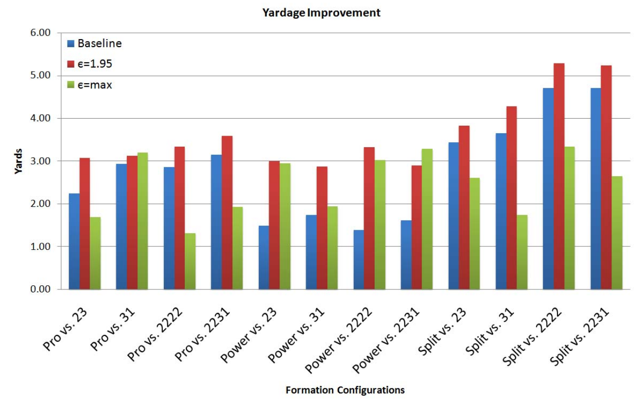


Figure 4: Comparison of play switch selection methods. Our play switch method (shown in red) outperforms both baseline Rush offense (blue) and a greedy play switch metric (green).

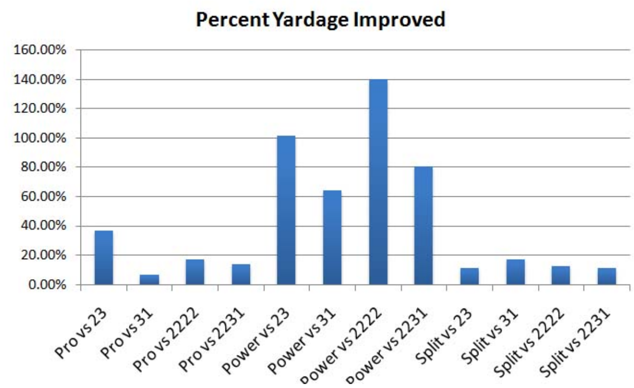


Figure 5: The play-yardage gain over baseline Rush offense yielded by our play switch strategy.

about .2 to .4 yards more than the gains in the baseline sample. In 6 we see all the split configurations do quite well; this is unsurprising given our calculations of the best response. However, when the threshold is not in use and the plays are allowed to change regardless of current yardage, the results are drastically reduced. The reason seems to be associated player miscoordinations accidentally induced by the play switch; by maximizing the play similarity simultaneously, the possibility of miscoordinations is reduced. Figure 5 shows yardage gained by the best play switch strategy over the Rush baseline offense. Power vs. 23 experiences the greatest enhancement and Split vs. 31 the least. It is interesting to note Split formations in the baseline performed best and improved the least while the Power formations performed the worst in the baseline and improved the most. This indicates an inversely proportional expected gain by the algorithm.

To evaluate the subgroup switching, we ran the simulation over all three groups and compared them to the baseline yardage gained and the results of total play switch. Test

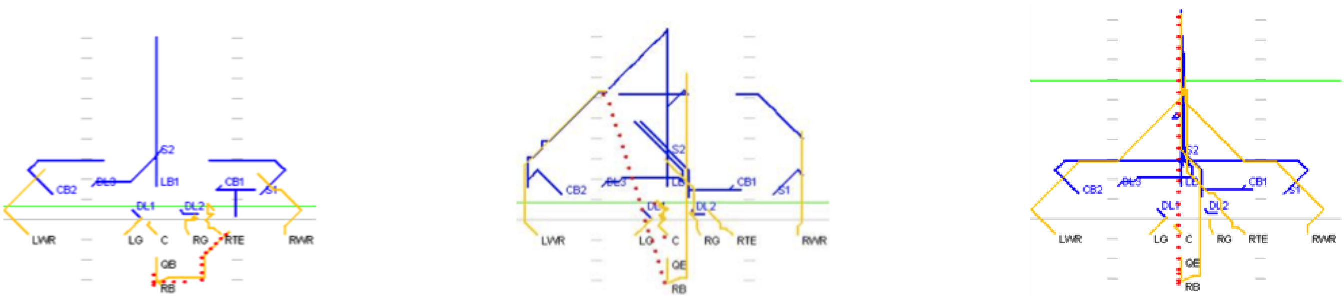


Figure 3: Subgroup switching

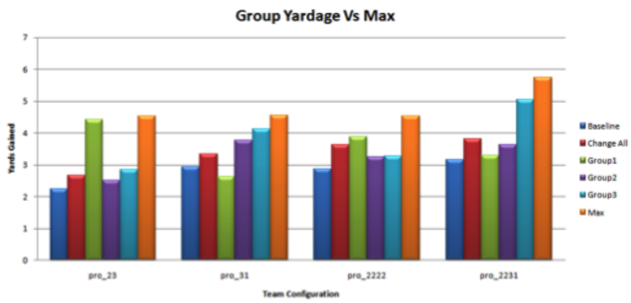


Figure 6: Comparison of subgroup and total play switching

results clearly indicated the best subgroup switch (consistently Group 1) produced greater gains than the total change, which still performed better than the baseline. Figure 2 is a side-by-side comparison of the results. We also compared the results to the yardage gained if the team had initially chosen the best response play (the play that on average results in the greatest yardage gain) for that formation. Early play recognition combined with subgroup switching yields the best results, assuming no oracle knowledge of the other team's intentions prior to run-time.

Conclusion

In this paper, we present an approach for early, accurate recognition of defensive plays in the Rush 2008 football simulator. We demonstrate that a multi-class SVM classifier trained on spatio-temporal game traces can enable the offense to correctly anticipate the defense's play by the third time step. Using this information about the defense's intent, our system evaluates the competitive advantage of executing a play switch based on the potential of other plays to improve the yardage gained and the similarity of the candidate plays to the current play. Our play switch selection mechanism outperforms both the built-in Rush offense and a greedy yardage-based switching strategy, increasing yardage while avoiding the miscoordinations accidentally induced by the greedy strategy during the transition from the old play to the new one. Additionally, we demonstrate that limiting the play switch to a subgroup of key players further improves performance.

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